

## Math Review Lab

### Algebra

#### Exponents:

$$a^n a^m = a^{n+m}$$

$$(ab)^n = a^n b^n$$

$$(a^n)^m = a^{nm}$$

$$\log(a^n) = n \log(a)$$

#### Binomial

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k \quad \text{where} \quad \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

#### Solving for zeros, quadratic formula

$$a x^2 + b x + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equation of a line:  $y = m x + b$

$m$  is the slope of the line

$b$  is the y-intercept

Slope =  $\Delta y / \Delta x$  approximated by  $(y_2 - y_1) / (x_2 - x_1)$

Finding the intersection of simple linear equations:

$$(1) y = m_1 x + b_1$$

$$(2) y = m_2 x + b_2$$

$$y_1 = y_2: m_1 x + b_1 = m_2 x + b_2$$

$$\rightarrow (m_1 - m_2) x = b_2 - b_1$$

$$\rightarrow x^* = (b_2 - b_1) / (m_1 - m_2)$$

$\rightarrow$  plug the value of  $x^*$  into either (1) or (2) to obtain  $y^*$

## Calculus (basic/simple)

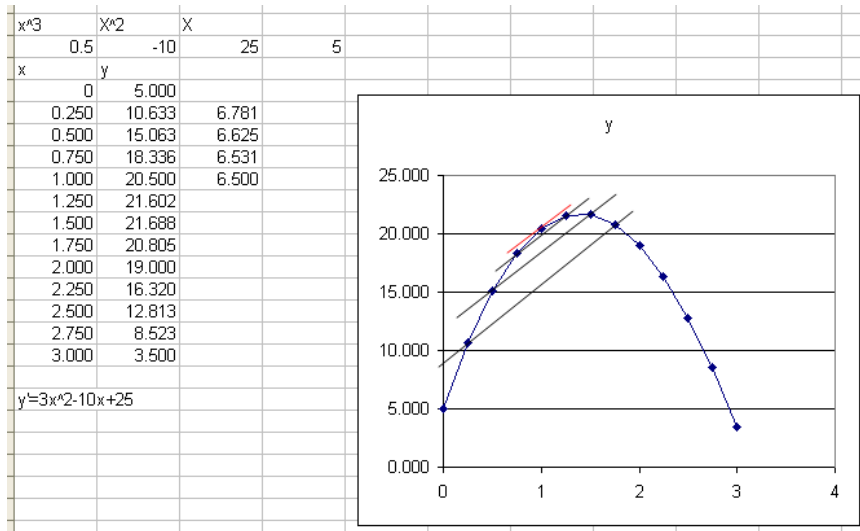
Derivatives: The slope of a continuous curve

$$\frac{dy}{dx} = f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$$

$$y = 0.5 x^3 - 10 x^2 + 25 x + 5$$

What is the slope at  $x=1$ ?

Approximate it:



$(y_2 - y_1) / (x_2 - x_1)$  varies depending on where you start.

The true slope occurs at a single point where a line is just tangent to the curve.

That slope is defined by the derivative of the function.

For polynomial functions, the derivative is straightforward:

Multiply each coefficient by the exponent and decrease the exponent by one.

$$y' = \frac{3}{2} x^2 - 20 x + 25 \quad \text{At } x=1, y' = 6.5$$

For a composite function:  $f(g(x))$ , the derivative is:  $f' \cdot g'$

$$\text{Example: } y = (x^3 + 4 x^2 + 3)^5$$

$$y' = 5(x^3 + 4 x^2 + 3)^4 (3x^2 + 8 x)$$

Slope is useful when searching for the maximum or minimum of a function. At each of these points, the slope will be zero, but you also need to check the second derivative to verify if it is a maximum ( $y'' < 0$ ) a minimum ( $y'' > 0$ ) or an inflection point ( $y'' = 0$ ).

$$\text{In the example, } y' = \frac{3}{2} x^2 - 20 x + 25 = 0$$

$$\text{Use the quadratic formula, } x = \frac{20 - \sqrt{400 - 300/2}}{3} = 1.396$$

$$y'' = 3 x - 20 \quad |_{x=1.396} \rightarrow -15.8 \text{ (negative indicates maximum)}$$

## Lab Assignments

1. Create numeric examples to demonstrate the validity of the exponential rules and test them in a spreadsheet. At least one example should use fractions.
2. Use a spreadsheet to chart two lines: (1)  $y = 5x - 100$ , (2)  $y = -7x + 100$ . Solve for the specific intersection point.
3. Given the curve:  $y = 3x^2 - 15x + 50$ 
  - a. Compute the derivative.
  - b. Find the minimum or maximum point.
  - c. Plot it in Excel.
4. Given the curve:  $y = 5K^{0.6}L^{0.4}$ 
  - a. Compute the two partial derivatives for K and L.
  - b. Use Excel to evaluate the function for K and L values from 0 to 15.
  - c. Plot the chart.